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Using Bayesian statistics to improve modal parameter estimates from an automatic OMA algorithm

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Abstract

Estimating damping is known to be notoriously inaccurate and the estimates typically have high variance. In this paper, Bayes theorem and conditional probability are utilized to reduce the variance of the damping estimates by accounting for multiple modal parameter estimates, i.e. poles and mode shape estimates. The modal parameters are estimated using the multi-reference Ibrahim Time Domain method, and an automatic operational modal analysis (AOMA) algorithm that utilizes histogram analysis has been used to automate the modal parameter estimation procedure. Data from a laboratory Plexiglass plate are used to investigate the proposed method. The results suggest that by applying Bayes theorem and conditional probability while accounting for multiple modal parameters improves the damping estimates by reducing the variance.

1 Introduction

The lifetime of flexible structures like, oil rigs, offshore wind turbines, long span bridges and skyscrapers, is governed by fatigue. Structural health monitoring (SHM), is the monitoring of the health of a structure, with the objective to quantify the remaining lifetime of the structure in question. Structural health monitoring is dating back to the mid nineties, and is, in principle based on sensor technology used to measure and characterize damage sensitive features, which adversely affect the performance of the system, see also [1] and [2]. One of the tools for diagnostics is operational modal analysis (OMA), which involves the estimation of modal parameters (i.e. frequency, damping and mode shape) for structures in operation [3]. A present day predominant loading scenario is fatigue, which is described as the weakening of materials due to repeated loading. The environmental forces (e.g. wind, waves and traffic) acting on the structures, discussed in this paper, include harmonic components. For any oscillatory system damping has the effect to reduce the amplitude of these harmonic components, and therefore, damping is an important measure when assessing the fatigue lifetime of structures. However, damping is difficult to estimate, and the estimates typically have high variance.

Probability is used to treat uncertainty in damping estimates. Inference as seen by the classical statistician, (or frequentist), quantifies probability in terms of population frequencies: given an event, the probability is the relative frequency under repeated observations, while for the Bayesian statistician, probability is the degree of belief that a given event will occur [4]. In other words, the classical statistician relies on the data observed in order to plausibly a given event, while the

Bayesian statistician can account for personal belief when quantifying the probability of the same event. The Bayesian approach offers an advantage over non-Bayesian ones for problems where it is non-trivial to construct an estimator for the parameters of interest. In particular problems that suffer from identification uncertainty that are necessary to quantify, i.e. they depend on underlying assumptions that should be accounted for. The assumption in this case is within a Bayesian framework the prior distribution, i.e. we assume or have some prior knowledge about the probability distribution of the sampled data we have obtained. The prior distribution and definition of such can also act as a drawback to Bayesian inference which is explained later on in this paper. For further reading on Bayesian inference, the reader is referred to [5].

The purpose of this paper is to utilize Bayesian inference given a prior distribution to obtain damping estimates with low variance from measurements on a Plexiglass plate obtained using an automated operational modal analysis algorithm.

2 Theory

In this Section, the Bayesian approach that is based upon conditional probability is described.

2.1 Bayesian inference

Let us consider a $Model$ and a set of $Data$ with the probability distributions, $P(Model)$ and $P(Data)$. Say, the probability distribution of the $Model$, $P(Model)$, is some prior knowledge we have, the prior. Then the probability that the $Model$ is true given some $Data$ is $P(Model|Data)$, is called the posterior. The likelihood is the probability that the $Data$ is true given some $Model$, i.e. $P(Data|Model)$. The probability distribution of the $Data$, $P(Data)$, is the evidence and acts as a normalization constant. Bayes' Rule, which was just described, is seen in Equation 1

$$\underbrace{P(Model|Data)}_{\text{Posterior}} = \frac{\overbrace{P(Data|Model)}^{\text{Likelihood}}\overbrace{P(Model)}^{\text{Prior}}}{\underbrace{P(Data)}_{\text{Evidence}}} \tag{1}$$

The posterior distribution is a joint probability density function conditional on modeling assumptions and available data. It can therefore be seen that in the special event where the same likelihood function in the Bayesian method is used and no prior information on the modal properties is used, the posterior estimates is equal to the maximum least square estimate.

3 Experiments

A vertically suspended Plexiglas plate was equipped with 35 uni-axial accelerometers that were mounted in a double symmetrical pattern and measuring in the same direction. The excitation was done using a pencil and the total measurement time correspond to 300 seconds, which is equivalent to approximately 42500 periods of the lowest natural natural frequency. The experimental setup may be seen in [6].

The measurements were post processed using an unbiased estimator to compute 1225 correlation functions using a blocksize of 512 samples. Subsequently the correlation functions were used as the input to the multi-reference Ibrahim Time Domain (MITD) method [7] and [8]. The modal parameters from the MITD method were quantified using an automated operational modal analysis algorithm that utilizes the statistical modeling of the stabilization diagram alongside a decision rule that is based on the modal assurance criterion, see [9]. The maximum model order was fixed at 60. However, the number of time lag values used in the correlation functions were varied from 40 to 440 in intervals of 2. This was done to obtain more estimates using the same data.

4 Results and discussion

Based on the approach described in the previous section we are able to extract a high number of damping estimates from little data in which we can construct a probability distribution of the damping ratio estimates. Many studies are reporting means and standard deviations of damping ratio estimates, e.g. [10], [11], and [12]. However, very few address the probability distribution of the damping ratio estimates. One study reports that the probability distribution is asymptotically normally distributed [13], while another study found that the modal parameters may be non-normally distributed, but it is generally following an asymptotically normal distribution [14].

In the following we will only consider the first mode (symmetrical bending around weak axis) for the Plexiglas plate. A total of 19587 damping ratio estimates were found. In Figure 1 a histogram of the estimates is presented. The number of bins are selected based on the Square-root Choice. Let N be the number of samples, then the number of bins are defined as No. Bins $= \sqrt{N}$. The total bin width is the distance from the lowest to the highest damping ratio estimate. The histogram is overlaid by a Laplace (double exponential) probability distribution. The fit seems reasonable, but it should be noted that there are a high concentration of damping ratio estimates at the tails and the distribution is not centered at the highest bin. An explanation for this is that the damping ratio estimates found when using few or many time lag values of the correlation functions have higher variance than the variance when using an intermediate number of time lag values. This can be seen in [15].

It is generally observed that when using stabilization diagrams, the frequency estimates tend to stabilize for increasing model order. This is also typical for the damping ratio estimates, but with a higher variance than for the frequency estimates. The probability distribution for the frequency estimates is, as for the damping ratio estimates, following a Laplace probability distribution. Furthermore, it was observed that the frequency and damping ratio estimates were uncorrelated, i.e. they were uncoupled from one another.

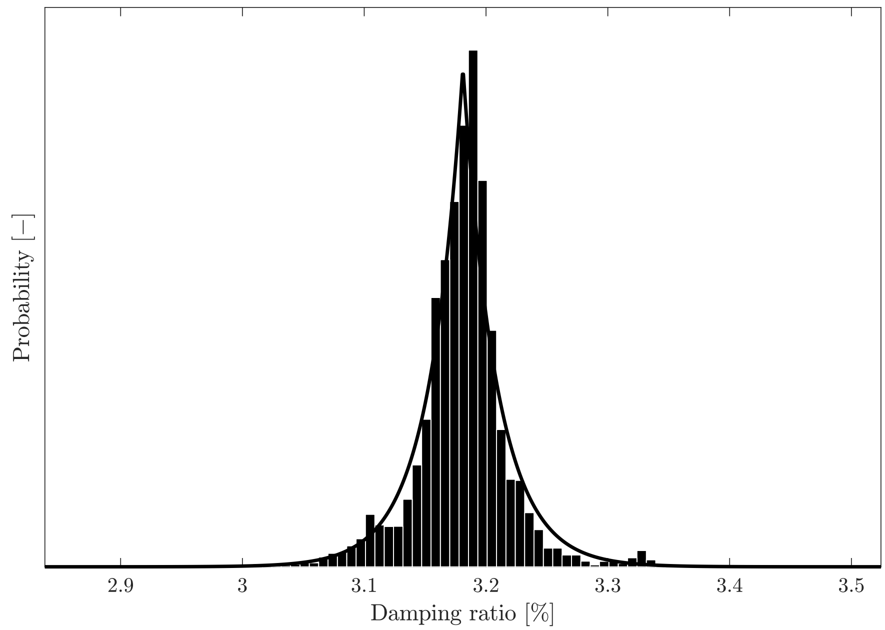

Figure 1: Histogram of 19587 damping ratio estimates for the first mode for the Plexiglass Plate. The solid black line represents a Laplace (double exponential) probability distribution

Within a Bayesian framework, the prior, as was mentioned in the introduction, has a backside. If the amount of data is small, the posterior will be predominated by the prior information, while if much data is used, the posterior will be governed by the likelihood information. Also the assumptions inherent to selecting the prior can compromise the Bayesian framework. In the following we will employ the probability distribution based on the 19587 damping ratio estimates as the prior distribution. By choosing only 4 damping ratio estimates to represent the likelihood distribution we can produce a posterior distribution. In Figure 2 we have the prior distribution shown as a solid black line. The variance of the prior distribution, and therefore also the variance of the damping ratio estimates is $\sigma^2_{prior} = 6.97e-08$. The likelihood distribution shown with a dashed black line has a variance of $\sigma^2_{likelihood} = 2.72e-08$, which is smaller than the variance of the prior distribution. By computing the posterior distribution, shown as the dotted black line, we end up having a variance of $\sigma^2_{posterior} = 1.96e-08$, which is lower than the variance of the prior and likelihood distributions.

It should be noted that the mean value is different for the three probability distributions. The prior that was constructed can be sensitive to small changes inherent from variations in environmental conditions, boundary conditions etc. It should therefore be used with care, and only when a proper baseline can be established.

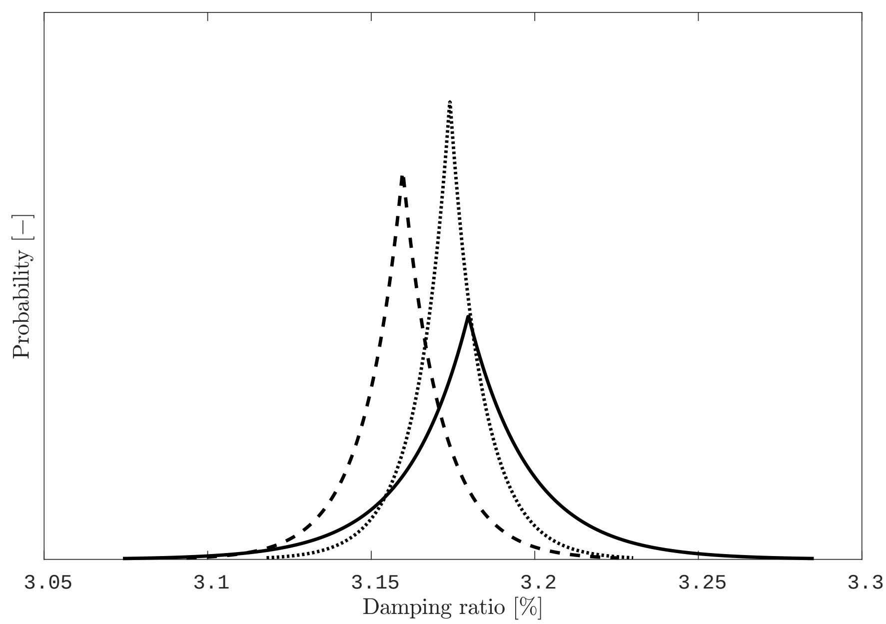

Figure 2: Bayesian inference. Solid black line - prior; dashed black line - likelihood; dotted black line - posterior

5 Conclusion

The damping ratio estimates for the first mode of a Plexiglas plate was shown to follow a Laplace (double exponential) probability distribution having a high concentration of data points at its mean value. Bayesian inference was employed by constructing a prior based on 19587 damping ratio estimates. The prior was subsequently used to find the posterior from a small sample of damping ratio estimates, the likelihood. The result showed that a lower variance was obtained. It should be stressed that the slightest change in environmental conditions, boundary conditions etc. can affect the prior.

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